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LETTER TO THE EDITOR

Quantum measurement as dissipation in chaotic atomic systems

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Abstract. It is argued through examples that for quantum non-linear Hamiltonians for atomic systems the main effect of measurement is the introduction of dissipation. In the dissipative case quantum measurements can be performed which lead to no disturbance of the system.

There has been much recent interest in chaotic and aperiodic behaviour in quantum systems [1]. Questions on the role of measurement and back reaction on the system being measured in such systems [2] have occasionally been raised. However, the analysis of quantum measurement, which has had a resurgence of interest, has often concentrated on epistemological questions. These issues have been discussed in the context of simple linear systems. We will, however, take a pragmatic point of view here and show some effects of measurement, which we believe are fairly general, on a large class of quantum mechanical systems (including non-linear chaotic ones). Quantum mechanical systems liable to experimental test are usually atomic, and so we shall restrict our attention in this letter to such systems. Moreover these systems can be non-dissipative or dissipative. If ρ is the density matrix of the system then the dynamics has the form

$$\dot{\rho}=\frac{1}{\mathrm{i}\hbar}[H,\rho]+\Lambda\rho \tag{1}$$

where H is the Hamiltonian for the system and Λ is a Liouvillian operator which is non-zero when there is dissipation. In many theoretical analyses of chaos the atomic systems are taken to have two states. We will, consequently, also for the measurement situation, consider a two-state atom A interacting with an arbitrary system S through some general Hamiltonian H. For the time being Λ will be taken to be zero. Before proceeding it will be convenient to introduce the irreducible tensor basis [3] for density matrix elements ρ^{J}_{m}

$$\rho=\sum_{\substack{\alpha,\beta\\ J,m}}\rho^{J,\alpha}_{m,\beta}T^{(J),\alpha}_{m,\beta} \tag{2}$$

where

$$T^{(J),\alpha}_{m,\beta}=\sum_{\substack{m',m''\\ J',J''}}(-1)^{J''-m''}\langle J',J'',m',-m''|J,m\rangle|J'm',\alpha\rangle\langle J''m'',\beta|.$$

Here in the ket $|J',m',\alpha\rangle$, (J',m') describes the angular momentum quantum numbers for A, and α the remaining quantum numbers to define the state of S. Moreover,

$\langle J', J'', m', -m''|J, m\rangle$ is the Clebsch-Gordan coefficient associated with the addition of angular momenta J' and J''.

In measurement theory it is customary to have a meter coupled to the system being measured. For an atomic system it is natural and convenient to have, as part of a meter, other levels of the atom which do not participate in the dynamics of the 'AS' system; there are no strictly two-level atoms. We will consider the 'meter' levels, M, of the atom to be also two in number. It is necessary to generalise (2) to

$$\rho = \sum_{\substack{\alpha,\beta \\ J,m}} \left(\rho^{J,\alpha}_{m,\beta}(AS)T^{(J),\alpha}_{m,\beta}(AS) + \rho^{J,\alpha}_{m,\beta}(M)T^{(J),\alpha}_{m,\beta}(M) + \rho^{J,\alpha}_{m,\beta}(AS, M)T^{(J),\alpha}_{m,\beta}(AS, M)\right) \tag{3}$$

where

$$T^{(J),\alpha}_{m,\beta}(AS) = \sum_{m',m''} (-1)^{J''-m''}\langle J', J'', m', -m''|J, m\rangle|J', m', \alpha, AS\rangle\langle J'', m'', \beta, AS|$$

$$T^{(J),\alpha}_{m,\beta}(AS, M) = \sum_{m',m''} (-1)^{J''-m''}\langle J', J'', m', -m''|J, m\rangle|J', m', \alpha, AS\rangle\langle J'', m'', \beta, M|$$

etc.

The labels M or AS on the bras and kets denote the angular momentum states corresponding to the 'meter' levels or the 'A' levels. We are, of course, interested in $J' = J'' = \frac{1}{2}$. It is then possible to show that, if we couple AS to M by a laser field, which to a good approximation can be taken as classical, the density matrix elements satisfy (for a closely related analysis, see [4])

$$\dot{\rho}^0_0(AS) = \gamma\rho^0_0(M) - \mathrm{i}e\rho^1_1(AS) + \mathrm{i}e^*\rho^1_1(AS) \tag{4a}$$

$$\dot{\rho}^0_0(M) = -\dot{\rho}^0_0(AS) \tag{4b}$$

$$\dot{\rho}^1_0(AS) = -\tfrac{1}{2}\gamma\rho^1_0(M) - \frac{\mathrm{i}}{\hbar}[H, \rho(AS)]^1_0 - \mathrm{i}e\rho^1_1(AS, M) + \mathrm{i}e^*\rho^{1*}_1(AS, M) \tag{4c}$$

$$\dot{\rho}^1_1(AS) = -\tfrac{1}{3}\gamma\rho^1_1(M) - \frac{\mathrm{i}}{\hbar}[H, \rho(AS)]^1_1 + \mathrm{i}e^*(\rho^0_0(AS, M)^* - \rho^1_0(AS, M)^*) \tag{4d}$$

$$\dot{\rho}^1_{-1}(AS) = -\tfrac{1}{3}\gamma\rho^1_{-1}(M) - \frac{\mathrm{i}}{\hbar}[H, \rho(AS)]^1_{-1} + \mathrm{i}e(\rho^0_0(AS, M) - \rho^1_0(AS, M)) \tag{4e}$$

$$\dot{\rho}^1_0(M) = -\gamma\rho^1_0(M) - \mathrm{i}e\rho^1_1(AS, M) + \mathrm{i}e^*\rho^{1*}_1(AS, M) \tag{4f}$$

$$\dot{\rho}^1_1(M) = -\gamma\rho^1_1(M) - \mathrm{i}e^*(\rho^{0*}_0(AS, M) + \rho^{1*}_0(AS, M)) \tag{4g}$$

$$\dot{\rho}^1_{-1}(M) = -\gamma\rho^1_{-1}(M) - \mathrm{i}e(\rho^0_0(AS, M) + \rho^1_0(AS, M)) \tag{4h}$$

$$\dot{\rho}^0_0(AS, M) = -\gamma_\perp\rho^0_0(AS, M) - \mathrm{i}e^*(\rho^1_{-1}(M) - \rho^1_{-1}(AS)) \tag{4i}$$

$$\dot{\rho}^1_0(AS, M) = -\gamma_\perp\rho^1_0(AS, M) - \mathrm{i}e^*(\rho^1_{-1}(M) + \rho^1_{-1}(AS, M)) \tag{4j}$$

$$\dot{\rho}^1_1(AS, M) = -\gamma_\perp\rho^1_1(AS, M) + \mathrm{i}e^*(\rho^0_0(M) - \rho^0_0(AS) - \rho^1_0(M) - \rho^1_0(AS)) \tag{4k}$$

$$\dot{\rho}^1_{-1}(AS, M) = -\gamma_\perp\rho^1_{-1}(AS, M). \tag{4l}$$

For clarity we have suppressed the α, β quantum numbers of S. Moreover we have defined

$$[H, \rho(AS)]^k_q = \mathrm{Tr}([H, \rho(AS)]\, T^{(k),\alpha}_{q,\beta}(AS)).$$

The quantity e is proportional to the Rabi frequency associated with the laser field. In order for the 'M' levels and the laser field to act as a meter we need the timescales $(1/\gamma, 1/\gamma_\perp)$ associated with them to be much faster than the dynamics being probed (which is governed by H). In this way the dynamics being studied can be followed in fine detail. Such fast timescales allow an adiabatic elimination of the $\rho(M)$ and $\rho(AS, M)$ matrix elements. We then obtain the equations

$$\dot\rho^1_0(AS) = -\frac{4}{3}\frac{|e|^2}{\gamma_\perp}\frac{1+\rho^1_0(AS)}{1+(6|e|^2/\gamma\gamma_\perp)} - \frac{\mathrm{i}}{\hbar}[H, \rho(AS)]^1_0 \tag{5a}$$

$$\dot\rho^1_1(AS) = \frac{2|e|^2}{\gamma_\perp}\rho^1_{-1}(AS) - \frac{\mathrm{i}}{\hbar}[H, \rho(AS)]^1_1 \tag{5b}$$

$$\dot\rho^1_{-1}(AS) = -\frac{2|e|^2}{\gamma_\perp}\rho^1_{-1}(AS) - \frac{\mathrm{i}}{\hbar}[H, \rho(AS)]^1_{-1}. \tag{5c}$$

Here $\rho^1_{\pm1}$ are coherences and ρ^1_0 is the population difference in the two states of A. One of the equations derivable from the adiabatic elimination conditions and used to obtain (5a)-(5c) is

$$\rho^1_1(AS, M) = \frac{\mathrm{i}e^*}{\gamma_\perp}(1+\rho^1_0(AS))\left(-1+\frac{6|e|^2}{\gamma\gamma_\perp(1+6|e|^2/\gamma\gamma_\perp)}\right). \tag{6}$$

This polarisation is related to the absorption of the laser light, and thus a measurement of the absorption constitutes a measurement of $\rho^1_0(AS)$. From (5a)-(5c) we see that the structure of the equations implies that the measurement causes dissipation. As a particularly simple example of this general result we can consider recent work [5] on chaotic 'Rabi' oscillations. In that work, for a system with the Hamiltonian

$$H = \hbar\omega_L(t)J_x + h\omega_\parallel J_z \tag{7}$$

in the absence of measurement, $\rho^1_0(t)$ was found to have irregular and aperiodic time evolution when the $\omega_L(t)$ time dependence was governed by two incommensurate frequencies which were also incommensurate with $\omega_\parallel$. In figure 2 we see the damping effect of measurement on ρ^1_0, in contrast to the behaviour in figure 1 where there is no measurement. (We define $s = 2|e|^2/\gamma_\perp$.) Although this is a specific (but natural) example of measurement, it holds for a large class of systems and illustrates in a particularly simple way that dissipation should be a primary effect of measurement, even in chaotic situations.

There are, of course, systems for which dissipation is an intrinsic part of the chaos [6] that is found. We may wonder whether the effect of measurement is just to add more dissipation to the existing dissipation in the system. The dissipative master equations, such as (1), are derived from a system evolving through a Hamiltonian H and also interacting with a heat bath. The standard Hamiltonian for the bath and the interaction of the bath and system is given by

$$\hbar\int \mathrm{d}\omega\, \omega a^+(\omega)a(\omega) + \frac{\mathrm{i}\hbar\gamma^{1/2}}{\sqrt{2\pi}}\int \mathrm{d}\omega(a^+(\omega)\sigma - \sigma^+ a(\omega))$$

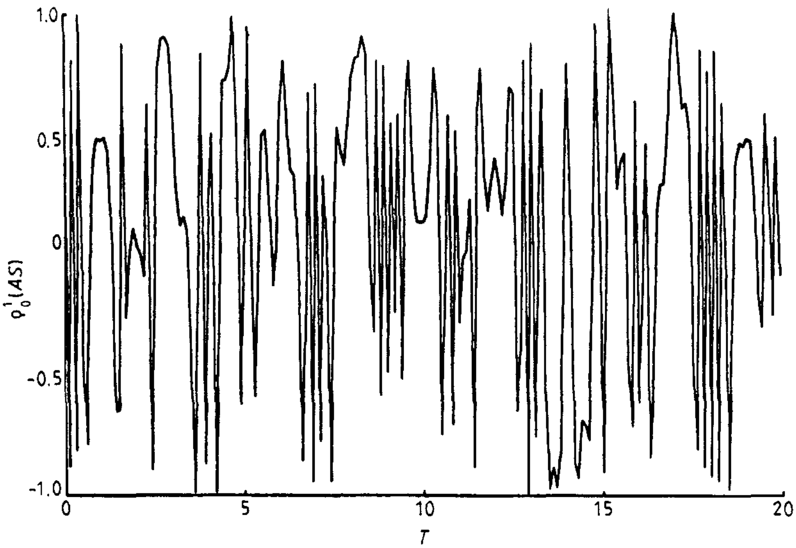

Figure 1. Driven two-state population difference with no measurement.

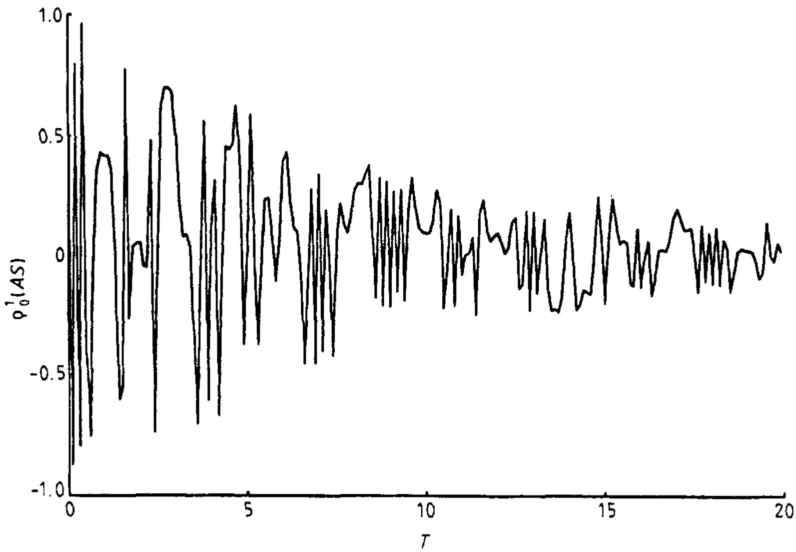

Figure 2. Driven two-state population difference with measurement ($s = 0.1$).

where $a(\omega)$, $a^{+}(\omega)$ are harmonic oscillator annihilation and creation operators satisfying

$$[a(\omega), a^{+}(\omega')] = \delta(\omega - \omega')$$

and σ is a system operator. The master equation can easily be shown to be equivalent to the set of quantum Langevin equations [7] for arbitary system operators σ'. For $t > t_0$

$$\dot{\sigma}' = -\frac{\mathrm{i}}{\hbar}[\sigma', H] - ([\sigma', \sigma^{+}](\tfrac{1}{2}\gamma\sigma + \gamma^{1/2}a_{\mathrm{in}}(t)) - (\tfrac{1}{2}\gamma\sigma^{+} + \gamma^{1/2}a_{\mathrm{in}}^{+}(t))[\sigma', \sigma]) \tag{8}$$

where

$$a_{\mathrm{in}}(t)=\frac{1}{(2\pi)^{1/2}}\int \mathrm{d}\omega\ \exp[-\mathrm{i}\omega(t-t_0)]a_0(\omega)$$

and $a_0(\omega)$ is $a(\omega)|_{t=t_0}$. In an analogous way for $t<t_1$ it can be shown that

$$\dot{\sigma}'=-\frac{\mathrm{i}}{\hbar}[\sigma', H]-([\sigma',\sigma^+](-\tfrac{1}{2}\gamma\sigma+\gamma^{1/2}a_{\mathrm{out}}(t))-(-\tfrac{1}{2}\gamma\sigma^++\gamma^{1/2}a^+_{\mathrm{out}}(t))[\sigma',\sigma]) \qquad (9)$$

where

$$a_{\mathrm{out}}(t)=\frac{1}{(2\pi)^{1/2}}\int \mathrm{d}\omega\ \exp[-\mathrm{i}\omega(t-t_1)]a_1(\omega).$$

It follows from (8) and (9) that

$$\sigma(t)=\gamma^{-1/2}(a_{\mathrm{out}}(t)-a_{\mathrm{in}}(t)). \qquad (10)$$

In this way, by monitoring the output into the bath we get a direct measurement of the system variable $\sigma(t)$. For atomic systems the $a(\omega)$ is the annihilation operator for free space photons of frequency ω. Hence by a simple reinterpretation of the Langevin equations for a quantum dissipative system we see directly that the dissipation present leads to a measurement of the system. No further dissipation mechanisms need be introduced.

In conclusion we find that quantum measurements can be performed on chaotic dissipative systems without affecting the system. For chaotic Hamiltonian systems, dissipation is introduced by the measurement.

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